Kite stability criteria

Derivation of the stability condition for single line kites

Nicolas Wadsworth (10.02) (retype 10.05)

In this note I show how I derive the equation

$$0 < d/L < AL\rho/M$$

as the condition for the lateral stability of a single line kite, drawing attention to the simplifying assumptions I make and the considerable approximations involved. Please see this article for original more general article on kite stability.

We think of a (box) kite with vertical surfaces, flown on a long line with multiple bridles, so the only rotation possible is about the direction of the kiteline. We assume:-

> All displacements are small (compared with the length of the kiteline),
> All sideways velocities are small (compared with the windspeed) and
> All rotations are small.

This means we consider only first order terms: the equations are linear.

We also assume the aerodynamic forces on the kite are a function only of its current sideways velocity, angle and rate of rotation and do not depend on its earlier motion. [Note that this is not really true. The kite is light enough to change its velocity, angle and rate of rotation quite a lot in the time the wind takes to travel the length of the kite, so the forces on the tail end in particular are seriously affected by what the kite was doing in the past. We ignore this, and hope it does not affect the qualitative result.]

You will also see that I tend to ignore numerical factors not too far from 1, or to be more precise, I hide them with the larger aerodynamic uncertainties in the terms "effective" side area, and "spread" of the side area. Those who wish to present these terms more explicitly can do so.
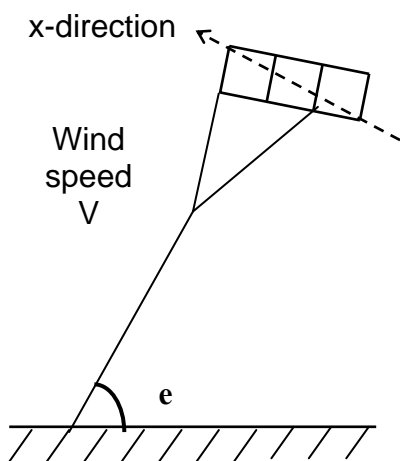

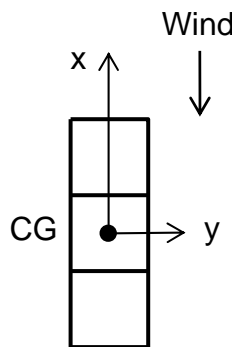**Fig.1**
Equilibrium Position

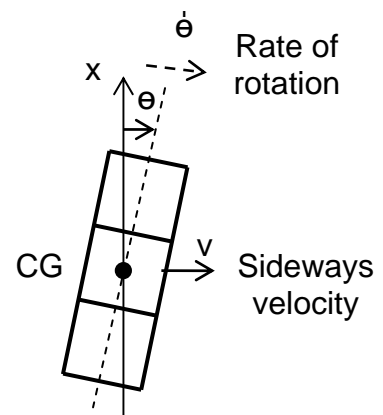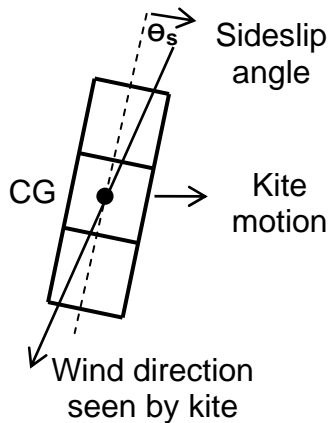**Fig.2 Equilibrium Position in plane perpendicular to kiteline**

Fig.3 After a disturbance

Fig.1 shows the kite in its equilibrium position at an angle of elevation **e** in a wind of speed **V**. Fig 2 shows the plane perpendicular to the kiteline with the y axis perpendicular to the wind. The long multiple bridles ensure that the only movements available to the kite are movements in the x and y directions and rotations about the normal to the plane, the direction of the kiteline. Motion in the x direction does not produce forces in the y direction or torques in the plane so the only variables we need to consider are v, θ and $\dot{\theta}$ as shown in Fig.3. We take the origin of the coordinates at the centre of mass of the kite, CG, and refer all movements, forces and torques to that point.

Aerodynamic forces

We first calculate the aerodynamic forces. These forces are produced by the wind blowing on the side panels of the kite as a result of the sideslipping of the kite as a whole or by rotation of the kite.

Take sideslip first. The kite is sideslipping if the wind is blowing against the sidepanels rather than directly along its axis (Fig.4). The sideslip velocity is **s** if the kite sees the wind as coming at a sideslip angle $\theta_s = s/V$ to the plane of the sidepanels, where **V** is the windspeed.

This wind at an angle θ_s to the sidepanel will produce a sideforce of $C_L.\tfrac{1}{2}\rho V^2$ per unit area, where C_L is the lift coefficient for a flat plane at an angle of incidence θ_s. The sideforce is thus $(C_L A'/2\theta_s).\rho Vs$ where A' is the actual side area. C_L for squareish flat panels seems to be about $2\theta_s$, so it is convenient to define an "effective" sidearea, A, equal to the real one multiplied by $C_L/2\theta_s$. The sideforce opposes the motion so it has the opposite sign to s. The sideforce due to a sideslip velocity s is thus $-A\rho Vs$. It is convenient to shorten this to **–Bs**,

In general the sideforce will not act at CG. If it is a distance d ahead of CG it will produce a torque about CG of –d.Bs. So: **a sideslip velocity s produces a sideforce –Bs and a torque –d.Bs, where B=AρV.** (see Fig.5)
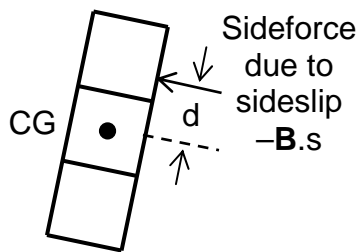

Fig.5 Sideforce due to sideslip

To find the sideslip velocity we need to add the contributions from each component of motion in Fig.3, $(v, \theta, \dot\theta)$. Firstly **a sideways velocity v on its own produces a sideslip velocity s=v.** Now consider rotation on its own. If the kite is rotating then the side area in front of the centre of rotation is moving to one side and being resisted by a force generated by the local sideslip. The side area behind the centre of rotation is moving the other way and being resisted by a force in the opposite direction. For a suitable centre of rotation these two forces will be equal so the rotation will produce a pure torque. The forces will be proportional to the rate of rotation, $\dot\theta$, and depend on ρ and V in the same way as that due to pure sideslip. We define a distance, L, the "spread" of the side area, such that the torque is given by $-L^2 B\dot\theta$, and **n** as the distance this centre of rotation is ahead of CG.

[In the oversimplified model where each element of side area behaves independently, L is the root-mean-square distance of the side area from the centre of rotation and **n=d**.]

If the kite is rotating at $\dot\theta$ about CG we can decompose the motion into a rotation $\dot\theta$ about the centre of rotation a distance **n** in front of CG and a sideways velocity of the whole kite of **n$\dot\theta$**. So :– **a rotation about CG of $\dot\theta$ produces a torque $-BL^2\dot\theta$ and contributes n$\dot\theta$ to the sideslip velocity.**

The only remaining thing to note is that if the kite is stationary but rotated at an angle θ to the x direction (Fig.3) the wind will be blowing on its sidepanels so it will be sideslipping. The sideslip velocity will be **V.sin e.θ** since the rotation axis (parallel to the kiteline) is at an angle e to the wind.

Dynamics

We are now ready to set up the equations of motion (ignoring gravity). The sideslip velocity is the sum of the three terms calculated so far.

$$s = v - V.\sin e.\theta + n\dot\theta \qquad\qquad ....(1)$$

This sideslip produces a sideforce –Bs giving

$$M\dot v = -Bs \qquad\qquad ....(2)$$

Where M is the mass of the kite.

The torque about CG is $-d.Bs -BL^2\dot{\theta}$, giving

$$Mk^2\ddot{\theta} = -d.Bs -BL^2\dot{\theta} \qquad \text{.... (3)}$$

Where Mk^2 is the moment of inertia of the kite about CG in the plane.

I am not proposing to solve equations (1), (2) and (3) explicitly. (Let me know if you get anything useful.) One can simulate them on a computer if one wishes. The basic behaviour is that if the kite is unstable the sideslip angle increases indefinitely and if it is stable the sideslip angle decreases to zero but the kite may be left at an angle to the wind and moving steadily across the sky. (We then rely on the ignored gravity to get it back to the vertical.)

The dividing line between stable and unstable behaviour occurs when the sideslip, **s**, becomes constant and finite. Equation (2) shows that $\dot{v}$ is then constant, so **v** increases linearly. It follows from (1) that θ must also increase linearly, so $\ddot{\theta}$ is zero.

Putting $\ddot{\theta} = 0$ in (3) gives

$$d.s = -L^2\dot{\theta} \qquad \text{.... (4)}$$

Differentiating (1) and putting $\dot{s}$ constant and $\ddot{\theta} = 0$ gives $\dot{v}=V.\sin e.\dot{\theta}$ and substituting for $\dot{v}$ from (2) gives
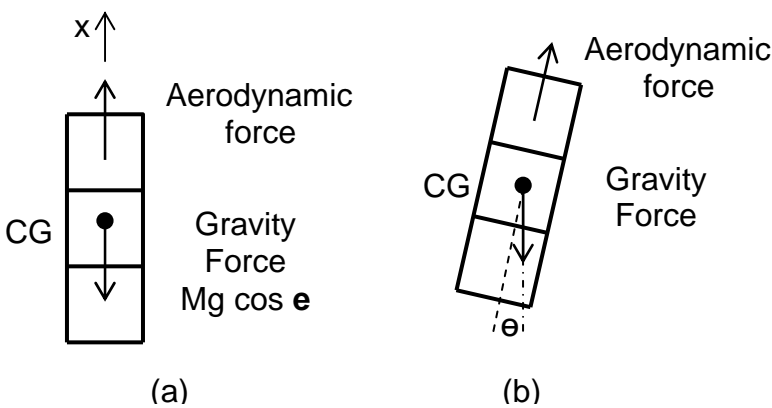
$$Bs/M = -V.\sin e.\dot{\theta} \qquad \text{.... (5)}$$

Comparing (4) and (5) gives $d/L^2 = B/MV.\sin e$, and replacing B by $A\rho V$ gives

$$d/L = AL\rho/(M.\sin e) \qquad \text{.... (6)}$$

as the limiting condition for stability.

When quoting this result I feel justified in dropping the sin e term which is likely to be 0.7 to 0.9. The uncertainties in A and L for a real kite are much greater. One can include it in the value for A or L if one wishes. In practice if you re-bridle the kite to fly at a different angle of elevation to try to see the effect of a change in **e** you will alter the kites attitude and hence the masking of the rear by the front. The resulting changes in A and L will be greater than the small change in sin **e**.

Fig,6 Forces in x-y plane (no sideslip or rate of rotation)

The effect of gravity

So far we have ignored the effect of gravity. Gravity has two effects. The major effect is to rule out zero or negative values of **d**, as I describe in "Why won't it fly" *(The Kiteflyer issue 91 pp7-12 April 2002)*. Basically, when the kite is a long way to one side of the vertical plane, so the kiteline is leaning sideways at an appreciable angle, gravity produces a steady sideways force (acting at CG) causing it to sideslip further away. It is necessary that the aerodynamic sideforce due to sideslip produces a torque to turn the nose up and so turn the kite back towards where it should be. In other words the sideforce due to sideslip must be ahead of that due to gravity, or

$$0 < d$$

This effect is negligible in the stability case we have been considering as the sideways movements are very small compared with the long kiteline assumed and the kite is never significantly away from the vertical plane. [Of course if the kiteline is very short this force can alter the behaviour and kites which are stable on a long line can be unstable when being launched on a very short line.]

3

However there is a second effect. Consider the forces in the plane when the kite is in the equilibrium position. (Fig.6a) There is a component of gravity in the plane, of Mg.cos e, pulling the kite in the –x direction. The kiteline force has no component in this plane (by definition) so the aerodynamic force must provide an equal and opposite force in the +x direction.

Now consider the kite rotated a (small) angle ϴ in the plane (and moving sideways to avoid sideslip). The wind relative to the kite is unaffected and the aerodynamic force in the plane is still along the kite length. The component of gravity in the plane is still the same but is now at an angle ϴ to the kite axis. (Fig. 6b) To a first order the component along the kite axis is still the same, so the fore-and-aft equilibrium is unaffected by the rotation, but there is an extra sideways component Mg cos **e**.ϴ acting at CG. Thus equation (2) becomes

$$M\dot{v} = -B.s + Mg\ cos\ e.\theta \qquad\qquad\qquad \textbf{. . . . (2A)}$$

The effect of this is that if after a disturbance the kite has been rotated, it will start to sideslip and the aerodynamic force due to sideslip, being ahead of CG, will turn the kite to reduce the rotation angle and turn it towards its original direction.

The equations of motion to be solved are now (1), (2A) and (3) and the analytical solution is even more complicated. Basically the solution is a damped (or overdamped) oscillatory approach to the equilibrium state if the kite is stable and an oscillation of increasing amplitude if the kite is unstable. The dividing line at the stability limit occurs when the damping is zero and the motion is an undamped sinusoidal oscillation.

One can set one of the variables to be a sinusoidal function of time and then use the equations to calculate the amplitudes and phases of the other variables and deduce the frequency and the value of d/L. It gets messy, so I leave it as "an exercise for the reader". Clearly since the limiting state is an oscillation one would expect the moment of inertia of the kite to be involved, also **n**, the distance of the effective centre of area from CG, and the strength of gravity relative to the aerodynamic forces.

The result I come up with for the limiting condition is

$$\frac{d}{L} = \frac{AL\rho}{M\ sin\ e} \Bigg/ \left\{ 1 + \frac{Mg}{A\rho V^2\ tan\ e} \cdot \left(\frac{k^2}{k^2 + nd + L^2} \right) \right\}$$

For small Mg this is the same as the original limit, as it should be. Since the kite must lift, ½ρV². (horizontal area) must be appreciably larger than Mg so the term in curly brackets is unlikely to be large. It just adds considerable complexity for little or no benefit. I have not noticed any behaviour which I can attribute to it so I treat is as one of the minor factors which are near 1.

So for practical purposes I use the formula:–

$$\textbf{0 < d/L <AL}\boldsymbol{\rho}\textbf{/M}$$

where –

> **d = distance sideforce is ahead of CG**
> **A = effective side area of kite**
> **L = spread of side area**
> **ρ = density of air**
> **M = mass of kite.**